Intelligence Modeling in Electromechanical Systems

www.imes.reapress.com

Intell. Model. Electromech. Syst. Vol. 2, No. 1 (2025) 16–31.

Paper Type: Original Article

Telecommunications Hypernetwork and Telecommunications SuperHypernetwork

Takaaki Fujita

Independent Researcher, Shinjuku, Shinjuku-ku, Tokyo, Japan; Takaaki.fujita060@gmail.com.

Citation:

Received: 07 July 2024 *Revised: 17 August 2024* *Accepted: 02 December 2024*	Fujita, T. (2025). Telecommunications hypernetwork and telecommunications superhypernetwork. *Intelligence modeling in electromechanical systems*, *2*(1), 16-31.

Abstract

A hypergraph generalizes the classical notion of a graph by allowing edges—called hyperedges—to connect more than two vertices simultaneously. A superhypergraph further extends this idea by introducing recursively nested powerset layers, thus enabling hierarchical and self-referential relationships among hyperedges. Graphs are widely used to represent networks. In this context, hypernetworks and superhypernetworks serve as the network analogues of hypergraphs and superhypergraphs, respectively.
In this paper, we focus on the concept of the Telecommunications Network. A Telecommunications Network enables the transmission of data, voice, and video among devices using wired or wireless communication technologies. We further examine the mathematical definitions, structural properties, and real-world examples of the *Telecommunications HyperNetwork* and the *Telecommunications SuperHyperNetwork*, which extend the classical Telecommunications Network to higher-order and hierarchical communication models.

Keywords: Superhypergraph, Hypergraph, Hypernetworks, Superhypernetworks, Telecommunications Network.

Corresponding Author: Takaaki.fujita060@gmail.com

1|Preliminaries

This section provides an overview of the fundamental concepts and definitions essential for the discussions in this paper. Throughout this paper, we consider only finite structures. Unless otherwise specified, all graphs are assumed to be simple, i.e., without multiple edges. For more detailed explanations and operational procedures, the reader is encouraged to consult the relevant references as needed.

1.1|SuperHyperGraph

We begin by presenting the definitions of Graph, HyperGraph, and SuperHyperGraph. In classical graph theory [1, 2], a hypergraph generalizes a traditional graph by allowing edges—called hyperedges—to connect more than two vertices [3]. This generalization enables the modeling of more complex relationships among elements, making hypergraphs highly applicable in diverse fields [4, 5, 6]. A *SuperHyperGraph* is a more advanced extension of the hypergraph model that incorporates recursively defined powerset structures into the conventional framework. This concept has been recently introduced and widely investigated in the literature [7, 8, 9, 10]. The formal definition is provided below. Unless otherwise noted, we assume throughout this paper that n is a nonnegative integer.

Definition 1.1 (Base Set)**.** A *base set* S is the foundational set from which complex structures such as powersets and hyperstructures are derived. It is formally defined as:

$$S = \{x \mid x \text{ is an element within a specified domain}\}.$$

All elements in constructs like $\mathcal{P}(S)$ or $\mathcal{P}_n(S)$ originate from the elements of S.

Definition 1.2 (Powerset)**.** The *powerset* of a set S, denoted $\mathcal{P}(S)$, is the collection of all possible subsets of S, including both the empty set and S itself. Formally, it is expressed as:

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}.$$

Definition 1.3 (n-th Powerset)**.** (cf.[11])

The n-th powerset of a set H, denoted $P_n(H)$, is defined iteratively, starting with the standard powerset. The recursive construction is given by:

$$P_1(H) = P(H), \quad P_{n+1}(H) = P(P_n(H)), \quad \text{for } n \geq 1.$$

Similarly, the n-th non-empty powerset, denoted $P_n^*(H)$, is defined recursively as:

$$P_1^*(H) = P^*(H), \quad P_{n+1}^*(H) = P^*(P_n^*(H)).$$

Here, $P^*(H)$ represents the powerset of H with the empty set removed.

Definition 1.4 (Hypergraph)**.** [12, 3] A *hypergraph* $H = (V(H), E(H))$ consists of:

- A nonempty set $V(H)$ of vertices.
- A set $E(H)$ of hyperedges, where each hyperedge is a nonempty subset of $V(H)$, thereby allowing connections among multiple vertices.

Unlike standard graphs, hypergraphs are well-suited to represent higher-order relationships. In this paper, we restrict ourselves to the case where both $V(H)$ and $E(H)$ are finite.

Definition 1.5 (n-SuperHyperGraph)**.** [13]

Let V_0 be a finite base set of vertices. For each integer $k \geq 0$, define the iterative powerset by

$$\mathcal{P}^0(V_0) = V_0, \quad \mathcal{P}^{k+1}(V_0) = \mathcal{P}(\mathcal{P}^k(V_0)),$$

where $\mathcal{P}(\cdot)$ denotes the usual powerset operation. An *n-SuperHyperGraph* is then a pair

$$\mathrm{SHT}^{(n)} = (V, E),$$

with

$$V \subseteq \mathcal{P}^n(V_0) \quad \text{and} \quad E \subseteq \mathcal{P}^n(V_0).$$

Each element of V is called an *n-supervertex* and each element of E an *n-superedge*.

Example 1.6 (2-SuperHyperGraph in a Telecommunications Setting)**.** Consider a small telecommunications operator with four individual base stations:

$$V_0 \;=\; \{\, B_1,\, B_2,\, B_3,\, B_4\},$$

where each B_i denotes a distinct cellular or microwave tower. We wish to model hierarchical groupings of these base stations up to two levels (i.e. $n = 2$).

Level 1 (Clusters of Base Stations). First, form two clusters of base stations based on geographic proximity or functional role:

$$C_1 \;=\; \bigl\{B_1,\, B_2\bigr\}, \qquad C_2 \;=\; \bigl\{B_3,\, B_4\bigr\}.$$

Each C_j is itself an element of the powerset $\mathcal{P}(V_0)$. These clusters might represent, for example, the group of towers serving "Downtown" (C_1) and the group serving "Suburban" (C_2).

Level 2 (Regions as Sets of Clusters). Next, form two distinct *2-supervertices* (i.e. elements of $\mathcal{P}^2(V_0)$) by grouping the clusters:

$$R_1 \;=\; \bigl\{\, C_1,\, C_2\bigr\}, \qquad R_2 \;=\; \{\, C_1\,\}.$$

Here:

- $R_1 \in \mathcal{P}^2(V_0)$ is the "Metro Region" automatically combining both clusters C_1 and C_2.
- $R_2 \in \mathcal{P}^2(V_0)$ is the "Core Subregion" containing only cluster C_1.

Thus our set of 2-supervertices is

$$V \;=\; \bigl\{\, R_1,\, R_2\bigr\} \;\subseteq\; \mathcal{P}^2(V_0).$$

Superedges Between Regions. Finally, we define a collection of *2-superedges* $E \subseteq \mathcal{P}^2(V_0)$. For instance:

$$E_1 \;=\; \bigl\{\, R_1,\, R_2\bigr\}, \quad E_2 \;=\; \bigl\{\, R_1\bigr\}.$$

Explicitly,

$$E \;=\; \{\, E_1,\, E_2\,\}, \quad E_1 = \{\,\{C_1, C_2\},\, \{C_1\}\}, \quad E_2 = \{\{\, C_1, C_2\,\}\}.$$

Each superedge E_k is itself a subset of $\mathcal{P}^2(V_0)$. In practical terms:

- $E_1 = \{\, R_1,\, R_2\}$ models a service linkage or inter-regional backhaul that connects the full Metro Region R_1 with its Core Subregion R_2.
- $E_2 = \{\, R_1\}$ represents an internal traffic-aggregation or monitoring relationship confined to the Metro Region R_1 alone.

Interpretation in Telecommunications.

- At the *base level* (V_0), each B_i is a physical tower serving end-users.
- At *Level 1*, each cluster C_j groups nearby towers for local load balancing or frequency coordination. For example, $C_1 = \{B_1, B_2\}$ might share a local routing hub.
- At *Level 2*, each 2-supervertex R_ℓ represents a higher-level administrative or operational region. For instance, $R_1 = \{C_1, C_2\}$ is the entire metropolitan service area, while $R_2 = \{C_1\}$ is the inner downtown zone.
- A superedge like $E_1 = \{R_1, R_2\}$ can model a dedicated backhaul link or guaranteed-quality service path between the downtown zone R_2 and the larger metro region R_1. Likewise, $E_2 = \{R_1\}$ might indicate an internal monitoring or multicast relationship confined to the metro-wide infrastructure.

Thus,

$$\mathrm{SHT}^{(2)} \;=\; \bigl(V,\, E\bigr)$$

with

$$V = \{\, R_1,\, R_2\}, \quad E = \{\, \{R_1, R_2\},\, \{R_1\}\},$$

constitutes a concrete 2-SuperHyperGraph that captures two levels of hierarchy—clusters of towers and regions containing those clusters—together with service-link relationships among them.

1.2|HyperNetwork and SuperhyperNetwork

A hypernetwork connects nodes via hyperedges, enabling multi-node interactions and weighted attributes for complex relationships. A superhypernetwork uses n-level nested sets as nodes and hyperedges, capturing hierarchical groupings and weighted associations. The definitions of HyperNetwork and SuperhyperNetwork are presented below (cf.[14]).

Definition 1.7 (Hypernetwork)**.** (cf.[14]) A *hypernetwork* is an ordered triple

$$H = \bigl(V,\, \mathcal{E},\, w\bigr)$$

where

- V is a nonempty finite set of *nodes*;
- $\mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$ is the set of *hyperedges*, each hyperedge $e \in \mathcal{E}$ being a nonempty subset of nodes (allowing multi-node interactions);
- $w\colon \mathcal{E} \to \mathbb{R}_{\geq 0}$ is a *weight or attribute function* on hyperedges (omitted if unweighted).

A *directed hypernetwork* may be defined by replacing $\mathcal{E} \subseteq \mathcal{P}(V)$ with a set of *ordered* tuples of nodes or by equipping each $e \in \mathcal{E}$ with a head-tail partition. One can further add a *node-labeling* $\ell_V : V \to L_V$ and a *hyperedge-labeling* $\ell_{\mathcal{E}} : \mathcal{E} \to L_{\mathcal{E}}$ to record types or properties.

Definition 1.8 (n-SuperHypernetwork)**.** (cf.[14]) Let V_0 be a finite base set of *nodes*. Define the n-th iterated powerset recursively by

$$\mathcal{P}^0(V_0) = V_0, \qquad \mathcal{P}^{k+1}(V_0) = \mathcal{P}\bigl(\mathcal{P}^k(V_0)\bigr) \quad (k \geq 0).$$

An *n-superhypernetwork* is a tuple

$$\mathcal{N}^{(n)} \;=\; \bigl(V,\; \mathcal{E},\; w\bigr)$$

where

- $V \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-supernodes*;
- $\mathcal{E} \subseteq \mathcal{P}^n(V_0)$ is a finite set of *n-superedges*, each superedge $e \in \mathcal{E}$ being a nonempty subset of V;
- $w\colon \mathcal{E} \to \mathbb{R}_{\geq 0}$ is an optional *weight function* assigning a nonnegative real weight (or confidence) to each superedge.

In other words, both vertices and hyperedges of the network are drawn from the n-th powerset of the base node set, capturing up to n levels of hierarchical grouping.

Example 1.9 (2-SuperHypernetwork in a Telecommunications Infrastructure)**.** Consider a simplified telecommunications operator with four base network devices:

$$V_0 \;=\; \{\, D_1,\, D_2,\, D_3,\, D_4\},$$

where each D_i represents an individual network element (e.g., a router or switch). We construct a 2-superhypernetwork $\mathcal{N}^{(2)} = (V, \mathcal{E}, w)$ to capture two levels of hierarchical grouping: devices $\to$ subnets $\to$ regions.

Level 1 (Subnets as 1-Supernodes). Form two subnets (each a subset of V_0) based on logical or geographic segmentation:

$$S_1 \;=\; \bigl\{D_1,\, D_2\bigr\}, \qquad S_2 \;=\; \bigl\{D_3,\, D_4\bigr\}.$$

Clearly, $S_1, S_2 \in \mathcal{P}(V_0)$. These subnets might correspond to, for instance, a "North Campus" LAN (S_1) and a "South Campus" LAN (S_2).

Level 2 (Regions as 2-Supernodes). Next, group these subnets into two higher-level regions (elements of $\mathcal{P}^2(V_0)$):

$$R_A = \{ S_1, S_2 \}, \qquad R_B = \{ S_1 \}.$$

Here:

- $R_A = \{ S_1, S_2 \} \in \mathcal{P}^2(V_0)$ represents the entire operator's metropolitan backbone, comprising both subnets S_1 and S_2.
- $R_B = \{ S_1 \} \in \mathcal{P}^2(V_0)$ represents a focused "North Campus Region" covering only subnet S_1.

Thus the set of 2-supernodes is

$$V = \{ R_A, R_B \} \subseteq \mathcal{P}^2(V_0).$$

2-Superedges Among Regions. We now define a collection of 2-superedges $\mathcal{E} \subseteq \mathcal{P}^2(V_0)$. For example:

$$E_1 = \{ R_A, R_B \}, \quad E_2 = \{ R_A \}.$$

Explicitly,

$$\mathcal{E} = \{ E_1, E_2 \}, \quad E_1 = \{\{S_1, S_2\}, \{S_1\}\}, \quad E_2 = \{\{S_1, S_2\}\}.$$

Each E_k is itself an element of $\mathcal{P}^2(V_0)$, i.e. a subset of the set of 2-supernodes V.

Weight Function (Link Capacity). Define the weight function $w\colon \mathcal{E} \to \mathbb{R}_{\geq 0}$ to represent, for instance, the aggregate inter-region bandwidth (in Gbps) or reliability metric:

$$w(E_1) = 10, \qquad w(E_2) = 5.$$

Here:

- $w(E_1) = 10$ Gbps indicates a high-capacity backhaul link between the full metropolitan backbone R_A and its "North Campus" sub-region R_B.
- $w(E_2) = 5$ Gbps represents an internal monitoring or multicast channel that exists only within the metropolitan backbone R_A itself.

Interpretation in a Real-World Operator Network.

- At the *base level* $V_0 = \{D_1, D_2, D_3, D_4\}$, each D_i is a physical network device (e.g., edge router or switch) serving end users.
- At *Level 1*, each 1-supernode $S_j \in \{S_1, S_2\}$ is a logical subnet grouping devices for local traffic aggregation, firewall enforcement, or VLAN segmentation.
- At *Level 2*, each 2-supernode $R_\ell \in \{R_A, R_B\}$ represents a higher-level region or data center cluster. For instance, R_A models the entire metropolitan network region, while R_B isolates the North Campus region for specialized services (e.g., research labs, enterprise customers).
- The 2-superedge $E_1 = \{R_A, R_B\}$ models a primary backbone circuit that carries aggregated traffic from the North Campus region R_B into the broader metropolitan backbone R_A. Its weight $w(E_1) = 10$ Gbps reflects the capacity of that circuit.
- The 2-superedge $E_2 = \{R_A\}$ indicates an internal redundancy or multicast-streaming relationship confined solely to R_A. Its weight $w(E_2) = 5$ Gbps could represent an intra-region synchronization channel or backup link capacity.

In summary, the 2-superhypernetwork

$$\mathcal{N}^{(2)} = \big(V, \mathcal{E}, w\big), \quad V = \{ R_A, R_B \}, \quad \mathcal{E} = \{ E_1, E_2 \}, \quad w(E_1) = 10,\ w(E_2) = 5,$$

captures two hierarchical levels—subnets S_j and regions R_ℓ—along with weighted 2-superedges that encode high-level connectivity and capacity across the operator's network infrastructure.

2|Results of This Paper

This section presents and explains the main results of this paper. Specifically, we examine the concepts of Telecommunications Network, Telecommunications HyperNetwork, and Telecommunications SuperHyperNetwork in detail.

2.1|Telecommunications Network

A Telecommunications Network enables the transmission of data, voice, and video among devices using wired or wireless communication technologies (cf.[15, 16, 17, 18, 19]). In this paper, we attempt to provide a formal mathematical definition of a Telecommunications Network, as described below. Please note that this is merely one example of a possible definition and does not claim to cover all real-world scenarios exhaustively.

Definition 2.1 (Telecommunications Network)**.** A *telecommunications network* is a quadruple

$$\mathcal{N} = (V, E, \kappa, \delta),$$

where

- V is a finite set of *nodes* (e.g. switches, routers, terminals);
- $E \subseteq V \times V$ is a set of directed *links*;
- $\kappa\colon E \to \mathbb{R}_{>0}$ is a *capacity function*, assigning to each link $(u, v) \in E$ its maximum data-rate $\kappa(u, v)$;
- $\delta\colon E \to \mathbb{R}_{\geq 0}$ is a *delay function*, assigning to each link $(u, v) \in E$ its propagation or transmission delay $\delta(u, v)$.

Moreover, given a set of traffic demands $\mathcal{D} = \{(s_i, t_i, d_i) \mid s_i, t_i \in V,\ d_i \in \mathbb{R}_{>0}\}$, a *routing* is a collection of flows $f_i\colon E \to \mathbb{R}_{\geq 0}$ satisfying:

$$\sum_{(u,v)\in E} f_i(u, v) - \sum_{(v,w)\in E} f_i(v, w) = \begin{cases} d_i, & v = s_i, \\ -d_i, & v = t_i, \\ 0, & \text{otherwise}, \end{cases} \qquad \forall\, v \in V,$$

and the *capacity constraints*

$$\sum_i f_i(u, v) \ \leq\ \kappa(u, v) \quad \forall\, (u, v) \in E.$$

Example 2.2 (Telecommunications Network)**.** Consider a simple telecommunications network with three nodes:

$$V = \{A,\ B,\ C\},$$

where:

- A is a source switch,
- B is an intermediate router,
- C is a destination terminal.

The directed links E and their associated capacity κ and delay δ functions are given by:

$$E = \{\, (A, B),\ (A, C),\ (B, C)\, \},$$

with

$$\begin{aligned} \kappa(A, B) &= 100 \quad (\text{Mbps}), & \delta(A, B) &= 2 \quad (\text{ms}), \\ \kappa(A, C) &= 50 \quad (\text{Mbps}), & \delta(A, C) &= 5 \quad (\text{ms}), \\ \kappa(B, C) &= 80 \quad (\text{Mbps}), & \delta(B, C) &= 1 \quad (\text{ms}). \end{aligned}$$

Thus, the capacity function $\kappa\colon E \to \mathbb{R}_{>0}$ and delay function $\delta\colon E \to \mathbb{R}_{\geq 0}$ are explicitly:

$$\kappa(A, B) = 100,\ \kappa(A, C) = 50,\ \kappa(B, C) = 80, \quad \delta(A, B) = 2,\ \delta(A, C) = 5,\ \delta(B, C) = 1.$$

Suppose there is a single traffic demand:

$$\mathcal{D} = \{\, (s_1, t_1, d_1) \,\} = \{\, (A,\, C,\, 30) \,\},$$

meaning we wish to send $d_1 = 30$ Mbps from $s_1 = A$ to $t_1 = C$.

A possible routing consists of two flows $f_1 \colon E \to \mathbb{R}_{\geq 0}$, split over both available paths $A \to B \to C$ and $A \to C$. For instance:

$$f_1(A,B) = 20, \quad f_1(B,C) = 20,$$
$$f_1(A,C) = 10, \quad f_1(B,A) = 0, \;\; f_1(C,\cdot) = 0.$$

Here:

- Twenty Mbps of the demand travels along $A \to B \to C$:

  $$f_1(A,B) = f_1(B,C) = 20,$$

  and

- Ten Mbps travels directly over $A \to C$:

  $$f_1(A,C) = 10.$$

This routing satisfies the flow-conservation constraints at each node $v \in V$:

$$\sum_{(u,v)\in E} f_1(u,v) \;-\; \sum_{(v,w)\in E} f_1(v,w) = \begin{cases} +30, & v = A, \\ -30, & v = C, \\ 0, & v = B, \end{cases}$$

and it also respects the capacity constraints on each link:

$$f_1(A,B) = 20 \;\leq\; \kappa(A,B) = 100, \quad f_1(A,C) = 10 \;\leq\; \kappa(A,C) = 50, \quad f_1(B,C) = 20 \;\leq\; \kappa(B,C) = 80.$$

Finally, the total end-to-end delay for each path is:

$$\text{Path } A \to B \to C: \quad \delta(A,B) + \delta(B,C) \;=\; 2 + 1 = 3 \text{ ms},$$
$$\text{Path } A \to C: \quad \delta(A,C) = 5 \text{ ms}.$$

Hence, this concrete example illustrates a telecommunications network $\mathcal{N} = (V, E, \kappa, \delta)$, a traffic demand $\mathcal{D}$, and a valid routing f_1 that obeys both flow-conservation and capacity constraints, while quantifying per-link delays and overall path latency.

2.2|Telecommunications HyperNetwork

We attempt to provide a formal mathematical definition of a Telecommunications HyperNetwork, as described below.

Definition 2.3 (Telecommunications HyperNetwork)**.** Let V be a finite set of *nodes* (e.g. routers, switches, terminals). A *telecommunications hypernetwork* is a quadruple

$$\mathcal{H} = \big(V,\; \mathcal{E},\; \kappa,\; \delta\big)$$

where

- $\mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$ is the set of *hyperlinks*, each $e \in \mathcal{E}$ representing a multi-node communication group;
- $\kappa \colon \mathcal{E} \to \mathbb{R}_{>0}$ assigns to each hyperlink its *aggregate capacity* $\kappa(e)$;
- $\delta \colon \mathcal{E} \to \mathbb{R}_{\geq 0}$ assigns to each hyperlink its *delay* $\delta(e)$.

Data flow on hyperlink e can simultaneously reach all nodes in e, modeling multicast or broadcast links in a telecommunications setting.

Example 2.4 (Enterprise Video Conferencing as a Telecommunications HyperNetwork)**.** Video Conferencing is a technology that enables real-time audio and visual communication between people in different locations via the internet (cf.[20, 21, 22]). Consider an organization with four conference-room video endpoints:

$$V = \{\text{Room}_A,\ \text{Room}_B,\ \text{Room}_C,\ \text{Room}_D\}.$$

They use a multicast server to deliver a video stream simultaneously to subsets of rooms. We model this as a telecommunications hypernetwork $\mathcal{H} = (V, \mathcal{E}, \kappa, \delta)$ where the set of hyperlinks is

$$\mathcal{E} = \Big\{\{\text{Server}, \text{Room}_A, \text{Room}_B\},\ \{\text{Server}, \text{Room}_B, \text{Room}_C, \text{Room}_D\},\ \{\text{Server}, \text{Room}_A, \text{Room}_D\}\Big\},$$

each hyperlink modeling one multicast group. For example:

- $e_1 = \{\text{Server}, \text{Room}_A, \text{Room}_B\}$ is a small-group meeting link;
- $e_2 = \{\text{Server}, \text{Room}_B, \text{Room}_C, \text{Room}_D\}$ is a town-hall broadcast;
- $e_3 = \{\text{Server}, \text{Room}_A, \text{Room}_D\}$ is a two-site workshop link.

We assign capacities (in Mbps) and one-way delays (in ms):

$$\kappa(e_1) = 100, \quad \delta(e_1) = 20; \quad \kappa(e_2) = 500, \quad \delta(e_2) = 50; \quad \kappa(e_3) = 150, \quad \delta(e_3) = 25.$$

A single video stream of rate $r \leq \kappa(e_i)$ sent over hyperlink e_i reaches all rooms in e_i simultaneously, modeling real-time multicast. For instance, sending $r = 80$ Mbps over e_1 delivers the stream to Room_A and Room_B with 20 ms latency, while sending $r = 300$ Mbps over e_2 delivers to Room_B, Room_C, and Room_D with 50 ms latency.

Example 2.5 (IoT Firmware Distribution as a Telecommunications HyperNetwork)**.** IoT Firmware is embedded software in IoT devices that controls hardware functions, connectivity, and data processing for remote communication (cf.[23, 24]). Consider an IoT deployment with a central server and four sensor clusters:

$$V = \{\text{Server, Cluster}_1,\ \text{Cluster}_2,\ \text{Cluster}_3,\ \text{Cluster}_4\}.$$

Firmware updates are pushed via multicast links that reach multiple clusters simultaneously. We model this as the hypernetwork $\mathcal{H} = (V, \mathcal{E}, \kappa, \delta)$ with hyperlinks

$$\mathcal{E} = \Big\{e_1 = \{\text{Server, Cluster}_1,\ \text{Cluster}_2\},\ e_2 = \{\text{Server, Cluster}_2,\ \text{Cluster}_3\},$$
$$e_3 = \{\text{Server, Cluster}_3,\ \text{Cluster}_4\},\ e_4 = \{\text{Server, Cluster}_1,\ \text{Cluster}_4\}\Big\}.$$

Each hyperlink e_i represents a multicast channel from the server to the indicated clusters. We assign:

$$\kappa(e_1) = 50 \text{ Mbps}, \quad \delta(e_1) = 30 \text{ ms}; \quad \kappa(e_2) = 40 \text{ Mbps}, \quad \delta(e_2) = 25 \text{ ms};$$
$$\kappa(e_3) = 60 \text{ Mbps}, \quad \delta(e_3) = 35 \text{ ms}; \quad \kappa(e_4) = 30 \text{ Mbps}, \quad \delta(e_4) = 20 \text{ ms}.$$

To distribute a firmware image of size 200 Mb, the server selects a multicast link e_i with $\kappa(e_i) \geq 200$. For instance, using e_3 (capacity 60 Mbps) requires at least $\lceil 200/60 \rceil = 4$ successive transmissions, each incurring a 35 ms delay, to update Cluster_3 and Cluster_4. Alternatively, one can split targets: first send via e_1 to clusters 1 and 2, then via e_3 for clusters 3 and 4. This hypernetwork model enables optimized selection of multicast groups to balance capacity constraints and delay.

Theorem 2.6 (Generalization of Telecommunications Network)**.** *Every classical telecommunications network $\mathcal{N} = (V, E, \kappa_2, \delta_2)$, where $E \subseteq V \times V$, embeds into a telecommunications hypernetwork $\mathcal{H} = (V, \mathcal{E}, \kappa, \delta)$ by*

$$\mathcal{E} = \big\{\{u, v\} : (u, v) \in E\big\}, \quad \kappa(\{u, v\}) = \kappa_2(u, v), \quad \delta(\{u, v\}) = \delta_2(u, v).$$

Proof: Define a mapping $\phi\colon E \to \mathcal{E}$ by $\phi((u, v)) = \{u, v\}$. Since each pair $\{u, v\}$ is nonempty and distinct, $\mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$. Setting $\kappa(\{u, v\}) = \kappa_2(u, v)$ and $\delta(\{u, v\}) = \delta_2(u, v)$ preserves capacities and delays. Thus $\mathcal{N}$ is realized as the special case of $\mathcal{H}$ in which all hyperlinks have cardinality two. □

Theorem 2.7 (HyperNetwork Structure)**.** *A telecommunications hypernetwork $\mathcal{H} = (V, \mathcal{E}, \kappa, \delta)$ carries the structure of a hypernetwork in the sense of Definition 2.7: the pair $\big(V, \mathcal{E}\big)$ is a hypergraph on which we have added weight functions κ, δ.*

Proof: By construction, $\mathcal{E} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$, so $(V, \mathcal{E})$ is a hypergraph. The functions κ and δ assign nonnegative real weights to each hyperedge. Hence $\mathcal{H}$ is precisely a weighted hypernetwork as per Definition 2.7. □

Theorem 2.8 (Routing via Hyperlinks)**.** *In a telecommunications hypernetwork $\mathcal{H} = (V, \mathcal{E}, \kappa, \delta)$, any traffic demand (S, T, d) with $S, T \subseteq V$ can be satisfied by flows $f : \mathcal{E} \to \mathbb{R}_{\geq 0}$ satisfying*

$$\sum_{e \ni v} f(e) - \sum_{e \ni v} f(e) = \begin{cases} d, & v \in S, \\ -d, & v \in T, \\ 0, & \textit{otherwise}, \end{cases}$$

subject to capacity constraints $\sum_{S,T \subseteq e} f(e) \leq \kappa(e)$.

Proof: One treats each hyperedge e as a shared multicast link: flow $f(e)$ leaves any source node in $S \cap e$ and is received by all sink nodes in $T \cap e$. By enforcing the flow-conservation balances above and ensuring $\sum_e f(e) \leq \kappa(e)$, one generalizes classical max-flow formulations to the hypernetwork setting. □

Theorem 2.9 (Delay Aggregation)**.** *For any path of hyperlinks $e_1, e_2, \ldots, e_k$ with $e_i \cap e_{i+1} \neq \emptyset$, the end-to-end delay satisfies*

$$\Delta = \sum_{i=1}^{k} \delta(e_i).$$

Proof: Because a data packet traverses each hyperlink e_i in sequence, accumulating delay $\delta(e_i)$, the total delay is the sum. Overlaps $e_i \cap e_{i+1}$ ensure connectivity of the path. □

2.3|Telecommunications n-SuperHyperNetwork

We attempt to provide a formal mathematical definition of a Telecommunications n-SuperHyperNetwork, as described below.

Definition 2.10 (Telecommunications n-SuperHyperNetwork)**.** Let V_0 be a finite set of *nodes* (e.g. routers, switches, terminals). For a fixed integer $n \geq 1$, let

$$\mathcal{P}^n(V_0) \;=\; \underbrace{\mathcal{P}\big(\mathcal{P}(\cdots \mathcal{P}(V_0) \cdots)\big)}_{n \text{ iterated powersets}}.$$

A *telecommunications n-superhypernetwork* is a quadruple

$$\mathcal{H}^{(n)} \;=\; \big(V,\; \mathcal{E},\; \kappa,\; \delta\big)$$

where

- $V \;\subseteq\; \mathcal{P}^n(V_0)$ is a finite set of *n-supernodes*;
- $\mathcal{E} \;\subseteq\; \mathcal{P}^n(V_0)$ is a finite set of *n-superhyperlinks*, each $e \in \mathcal{E}$ a nonempty subset of V;
- $\kappa : \mathcal{E} \to \mathbb{R}_{>0}$ assigns to each superhyperlink its aggregate *capacity*;
- $\delta : \mathcal{E} \to \mathbb{R}_{\geq 0}$ assigns to each superhyperlink its *delay*.

Data transmitted on a superhyperlink e reaches all contained supernodes simultaneously, modeling multicast at hierarchical levels.

Example 2.11 (Telecommunications 2-SuperHyperNetwork for Hierarchical Multicast)**.** Hierarchical Multicast is a scalable data transmission method that organizes multicast routing in layers to efficiently distribute data across networks (cf.[25, 26, 27]). Let the base set of physical routers be

$$V_0 = \{R_1,\, R_2,\, R_3\}.$$

The first iterated powerset

$$\mathcal{P}^1(V_0) = \big\{\{R_1, R_2\},\, \{R_1, R_3\},\, \{R_2, R_3\}\big\}$$

represents *subnets* of two routers. The second iterated powerset

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big)$$

is the set of all *clusters of subnets*. For our 2-superhypernetwork, choose the supernodes

$$V = \big\{C_A,\, C_B\big\} \subseteq \mathcal{P}^2(V_0),$$

where

$$C_A = \big\{\{R_1, R_2\},\, \{R_2, R_3\}\big\}, \quad C_B = \big\{\{R_1, R_3\},\, \{R_2, R_3\}\big\}.$$

Define two superhyperlinks in $\mathcal{E} \subseteq \mathcal{P}^2(V_0)$:

$$e_1 = \{\, C_A,\, C_B \,\}, \quad e_2 = \{\, C_A \,\}.$$

Assign aggregate capacities and delays by

$$\kappa(e_1) = 100 \text{ Mbps}, \quad \delta(e_1) = 40 \text{ ms}; \qquad \kappa(e_2) = 50 \text{ Mbps}, \quad \delta(e_2) = 20 \text{ ms}.$$

Thus the 2-superhypernetwork $\mathcal{H}^{(2)} = (V, \mathcal{E}, \kappa, \delta)$ models:

- *Supernodes* C_A, C_B each grouping two overlapping subnets,
- *Superhyperlink* e_1 connecting both clusters for a full-network multicast,
- *Superhyperlink* e_2 servicing only cluster C_A.

A multicast stream of rate $r \leq 100$ Mbps on e_1 reaches all subnets in both clusters with 40 ms latency, while a smaller regional update of rate $r \leq 50$ Mbps can be delivered to cluster C_A alone via e_2 in 20 ms. This hierarchical setup captures multicast at two levels of network grouping.

Example 2.12 (Telecommunications 3-SuperHyperNetwork for Multi-Tier IoT Update Distribution)**.** Let the base set of edge devices be

$$V_0 = \{\text{EN}_1,\ \text{EN}_2,\ \text{EN}_3\}.$$

Then

$$\mathcal{P}^1(V_0) = \big\{\{\text{EN}_1, \text{EN}_2\},\ \{\text{EN}_1, \text{EN}_3\},\ \{\text{EN}_2, \text{EN}_3\}\big\},$$

representing *local subnets*. Next,

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big)$$

is the set of *regional clusters* of subnets. Choose three clusters:

$$R_A = \{\{\text{EN}_1, \text{EN}_2\}, \{\text{EN}_2, \text{EN}_3\}\}, \quad R_B = \{\{\text{EN}_1, \text{EN}_3\}, \{\text{EN}_2, \text{EN}_3\}\}, \quad R_C = \{\{\text{EN}_1, \text{EN}_2\}, \{\text{EN}_1, \text{EN}_3\}\}.$$

Finally,

$$\mathcal{P}^3(V_0) = \mathcal{P}\big(\mathcal{P}^2(V_0)\big)$$

is the set of *global divisions* of regional clusters. We define two supernodes:

$$G_1 = \{R_A, R_B\}, \quad G_2 = \{R_B, R_C\}.$$

Thus our 3-superhypernetwork is $\mathcal{H}^{(3)} = (V, \mathcal{E}, \kappa, \delta)$ with

$$V = \{G_1,\, G_2\} \subseteq \mathcal{P}^3(V_0),$$

and two superhyperlinks:

$$e_1 = \{G_1, G_2\}, \quad e_2 = \{G_1\}.$$

Assign aggregate capacities and delays:

$$\kappa(e_1) = 10 \text{ Gbps}, \quad \delta(e_1) = 100 \text{ ms}; \qquad \kappa(e_2) = 4 \text{ Gbps}, \quad \delta(e_2) = 40 \text{ ms}.$$

A firmware image of size 2 GB can be multicast globally via e_1 in a single transmission (provided $2\,\text{GB} \leq 10\,\text{Gbps}$ within the delay bound of 100 ms), reaching all devices in both divisions G_1 and G_2. Alternatively, a smaller patch of size 1 GB can be sent to division G_1 alone via e_2, leveraging its 40 ms latency. This hierarchical 3-superhypernetwork model captures multi-tier multicast distribution across local subnets, regional clusters, and global divisions.

Example 2.13 (Telecommunications 3-SuperHyperNetwork for Global CDN Hierarchy)**.** Let the base set of Points of Presence (PoPs [28]) be

$$V_0 = \{\mathrm{POP_{NY}},\, \mathrm{POP_{LA}},\, \mathrm{POP_{LDN}}\}.$$

Then

$$\mathcal{P}^1(V_0) = \big\{\{\mathrm{POP_{NY}}, \mathrm{POP_{LA}}\},\ \{\mathrm{POP_{NY}}, \mathrm{POP_{LDN}}\},\ \{\mathrm{POP_{LA}}, \mathrm{POP_{LDN}}\}\big\}$$

represents *metro clusters*. Next,

$$\mathcal{P}^2(V_0) = \mathcal{P}\big(\mathcal{P}^1(V_0)\big)$$

is the set of *regional clusters* of metro clusters. Choose two:

$$R_{\mathrm{Americas}} = \{\{\mathrm{POP_{NY}}, \mathrm{POP_{LA}}\},\ \{\mathrm{POP_{LA}}, \mathrm{POP_{LDN}}\}\}, \quad R_{\mathrm{EMEA}} = \{\{\mathrm{POP_{NY}}, \mathrm{POP_{LDN}}\},\ \{\mathrm{POP_{LA}}, \mathrm{POP_{LDN}}\}\}.$$

Finally,

$$\mathcal{P}^3(V_0) = \mathcal{P}\big(\mathcal{P}^2(V_0)\big)$$

is the set of *global divisions* of regional clusters. Define two supernodes:

$$G_{\mathrm{Primary}} = \{R_{\mathrm{Americas}}, R_{\mathrm{EMEA}}\}, \quad G_{\mathrm{Backup}} = \{R_{\mathrm{EMEA}}\}.$$

Thus our 3-superhypernetwork is $\mathcal{H}^{(3)} = (V, \mathcal{E}, \kappa, \delta)$ with

$$V = \{G_{\mathrm{Primary}},\, G_{\mathrm{Backup}}\} \ \subseteq\ \mathcal{P}^3(V_0),$$

and two superhyperlinks:

$$e_1 = \{G_{\mathrm{Primary}}, G_{\mathrm{Backup}}\}, \quad e_2 = \{G_{\mathrm{Primary}}\}.$$

Assign aggregate capacities (in Tbps) and delays (in ms):

$$\kappa(e_1) = 5, \quad \delta(e_1) = 150; \qquad \kappa(e_2) = 2, \quad \delta(e_2) = 80.$$

A high-volume content distribution session (e.g. live video) of rate up to 5 Tbps sent over e_1 reaches both primary and backup divisions with 150 ms end-to-end latency. A lower-priority but latency-sensitive update (rate ≤ 2 Tbps) can be delivered exclusively to the primary division via e_2 in 80 ms. This hierarchical model captures global CDN routing across metro clusters, regional clusters, and global divisions.

Theorem 2.14 (Generalization of Classical Telecommunications Networks)**.** *Every classical telecommunications network $\mathcal{N} = (V_0, E_0, \kappa_0, \delta_0)$, with $E_0 \subseteq V_0 \times V_0$, embeds into a telecommunications n-superhypernetwork $\mathcal{H}^{(n)}$ by*

$$V \ = \ \big\{\eta_n(v) : v \in V_0\big\}, \quad \mathcal{E} \ = \ \big\{\{\eta_n(u), \eta_n(v)\} \ : \ (u, v) \in E_0\big\},$$

and

$$\kappa\big(\{\eta_n(u), \eta_n(v)\}\big) = \kappa_0(u, v), \quad \delta\big(\{\eta_n(u), \eta_n(v)\}\big) = \delta_0(u, v),$$

where $\eta_n(x)$ denotes the n-fold nested singleton $\{\{\cdots\{x\}\cdots\}\}$.

Proof: Define $\eta_n \colon V_0 \to \mathcal{P}^n(V_0)$ by $\eta_1(x) = \{x\}$ and $\eta_{k+1}(x) = \{\eta_k(x)\}$. Then each original node v becomes an n-supernode $\eta_n(v)$, and each original link (u, v) becomes the superhyperlink $\{\eta_n(u), \eta_n(v)\}$. By construction $\mathcal{E} \subseteq \mathcal{P}^n(V_0)$ and capacity/delay assignments agree. Hence $\mathcal{N}$ is realized as the special case of $\mathcal{H}^{(n)}$ with all superhyperlinks of size two. □

Theorem 2.15 (Generalization of Telecommunications HyperNetworks)**.** *Every telecommunications hypernetwork $\mathcal{H} = (V_1, \mathcal{E}_1, \kappa_1, \delta_1)$ embeds into an n-superhypernetwork $\mathcal{H}^{(n)}$ by*

$$V \ = \ \big\{\eta_n(v) : v \in V_1\big\}, \quad \mathcal{E} \ = \ \big\{\{\eta_n(e) : e \in E\} : E \in \mathcal{E}_1\big\},$$

with weights $\kappa(\{\eta_n(e) : e \in E\}) = \kappa_1(E)$ and $\delta(\{\eta_n(e) : e \in E\}) = \delta_1(E)$.

Proof: Let η_n be as above extended to subsets of V_1 by $\eta_n(E) = \{\eta_n(v) : v \in E\}$. Then each hyperedge $E \subseteq V_1$ maps to an n-superhyperlink $\eta_n(E) \subseteq \mathcal{P}^n(V_0)$. Capacities and delays transfer accordingly. Thus $\mathcal{H}$ embeds into $\mathcal{H}^{(n)}$. □

Theorem 2.16 (n-SuperHyperNetwork Structure)**.** *A telecommunications n-superhypernetwork $\mathcal{H}^{(n)} = (V, \mathcal{E}, \kappa, \delta)$ is an n-superhyperstructure on the base set V_0: its supernodes and superhyperlinks are drawn from $\mathcal{P}^n(V_0)$, and the capacity/delay functions endow it with weighted superhypergraph structure.*

Proof: By definition, $V, \mathcal{E} \subseteq \mathcal{P}^n(V_0)$. The pair $(V, \mathcal{E})$ thus forms an n-superhypergraph, and κ, δ provide weights. Hence $\mathcal{H}^{(n)}$ satisfies the structure of an n-superhypernetwork as per Definition of n-superhypergraphs and weighted hypernetworks. □

Theorem 2.17 (Embedding of Lower-Level SuperHypernetworks)**.** *For each $0 \le k \le n$, the inclusion*

$$\iota_k\colon \mathcal{P}^k(V_0) \hookrightarrow \mathcal{P}^n(V_0), \quad X \mapsto \underbrace{\{\{\cdots\{X\}\cdots\}\}}_{n-k\ nestings},$$

identifies $\big(\mathcal{P}^k(V_0), \mathcal{E}_k, \kappa_k, \delta_k\big)$ as a sub-superhypernetwork of $\mathcal{H}^{(n)}$, where $\mathcal{E}_k = \mathcal{E} \cap \iota_k(\mathcal{P}^k(V_0))$ and weights restrict accordingly.

Proof: By construction, $\iota_k(X) \in \mathcal{P}^n(V_0)$ for all $X \in \mathcal{P}^k(V_0)$. Since $\mathcal{E} \subseteq \mathcal{P}^n(V_0)$, the intersection $\mathcal{E}_k = \mathcal{E} \cap \iota_k(\mathcal{P}^k(V_0))$ consists exactly of those n-superhyperlinks that arise from k-level objects. Restricting κ, δ to $\mathcal{E}_k$ makes $\big(\iota_k(\mathcal{P}^k(V_0)), \mathcal{E}_k, \kappa|_{\mathcal{E}_k}, \delta|_{\mathcal{E}_k}\big)$ itself an n-superhypernetwork. Because ι_k is injective and respects the nested-powerset structure, this copy is isomorphic to the k-superhypernetwork on $\mathcal{P}^k(V_0)$. □

Theorem 2.18 (Flattening to a HyperNetwork)**.** *There is a natural "flattening" map*

$$\pi\colon \mathcal{P}^n(V_0) \longrightarrow \mathcal{P}(V_0),$$

defined by iterated union $\pi(X) = \bigcup\big(\cdots \bigcup X \cdots\big)$. Under π, $\mathcal{E}$ is sent to a set of hyperedges in $\mathcal{P}(V_0)$, and $(V_0, \pi(\mathcal{E}), \kappa \circ \pi^{-1}, \delta \circ \pi^{-1})$ is a telecommunications hypernetwork.

Proof: Define π by $\pi_1(Y) = Y$ for $Y \subseteq V_0$ and $\pi_{k+1}(X) = \bigcup_{Y \in X} \pi_k(Y)$. Then $\pi = \pi_n$ maps each n-supernode $X \subseteq \mathcal{P}^{n-1}(V_0)$ to a subset of V_0. The image $\pi(\mathcal{E}) \subseteq \mathcal{P}(V_0)$ thus gives a set of hyperedges. Assigning capacities and delays by $\kappa'(e) = \sum_{E\colon \pi(E)=e} \kappa(E)$ and similarly for δ yields a weighted hypernetwork. Closure and weight-assignment verification are straightforward. □

Theorem 2.19 (Union of SuperHypernetworks)**.** *If $\mathcal{H}_1^{(n)} = (V_1, \mathcal{E}_1, \kappa_1, \delta_1)$ and $\mathcal{H}_2^{(n)} = (V_2, \mathcal{E}_2, \kappa_2, \delta_2)$ are two n-superhypernetworks on the same base V_0, then their union*

$$\mathcal{H}_\cup^{(n)} = (\, V_1 \cup V_2,\ \mathcal{E}_1 \cup \mathcal{E}_2,\ \kappa_\cup,\ \delta_\cup)$$

is also a telecommunications n-superhypernetwork, where $\kappa_\cup, \delta_\cup$ extend κ_i, δ_i by taking maximum weights on overlaps.

Proof: Since $V_1, V_2 \subseteq \mathcal{P}^n(V_0)$ and $\mathcal{E}_1, \mathcal{E}_2 \subseteq \mathcal{P}^n(V_0)$, their unions remain subsets of $\mathcal{P}^n(V_0)$. Defining $\kappa_\cup(E) = \max\{\kappa_1(E), \kappa_2(E)\}$ for $E \in \mathcal{E}_1 \cap \mathcal{E}_2$, and similarly for δ, ensures each superhyperlink has a well-defined capacity and delay. Hence $\mathcal{H}_\cup^{(n)}$ satisfies Definition 4.1. □

Theorem 2.20 (Connectivity Preservation)**.** *If the base network $\mathcal{N} = (V_0, E_0)$ is connected, then the induced n-superhypernetwork $\mathcal{H}^{(n)}$ is connected in the sense that for any two supernodes $X, Y \in V$, there exists a sequence of superhyperlinks*

$$E_1, E_2, \ldots, E_k \in \mathcal{E}$$

such that $X \cap E_1 \ne \emptyset$, $E_i \cap E_{i+1} \ne \emptyset$, and $E_k \cap Y \ne \emptyset$.

Proof: Connectivity of $\mathcal{N}$ means for any $u, v \in V_0$ there is a path $u = v_0, v_1, \ldots, v_m = v$ with $\{v_i, v_{i+1}\} \in E_0$. Under the embedding of Theorem 4.2, each node v_i lifts to a supernode $\eta_n(v_i)$ and each link $\{v_i, v_{i+1}\}$ to the superhyperlink $\{\eta_n(v_i), \eta_n(v_{i+1})\}$. These form a chain of superhyperlinks connecting $\eta_n(u)$ to $\eta_n(v)$. For arbitrary $X, Y \subseteq \mathcal{P}^n(V_0)$, pick $u \in \pi(X)$ and $v \in \pi(Y)$. The above yields a superpath between $\eta_n(u)$ and $\eta_n(v)$. Since $X \cap \{\eta_n(u)\} \ne \emptyset$ and $\{\eta_n(v)\} \cap Y \ne \emptyset$, concatenation gives the desired connectivity. □

Remark 2.21. Let V_0 be a finite base node set and write

$$\mathcal{P}^n(V_0) = \underbrace{\mathcal{P}\big(\mathcal{P}(\cdots\mathcal{P}(V_0)\cdots)\big)}_{n \text{ times}}.$$

Suppose $\mathcal{H}^{(n)} = (V, \mathcal{E}, \kappa, \delta)$ is a telecommunications n-superhypernetwork, and define the *flattening map*

$$\pi : \mathcal{P}^n(V_0) \longrightarrow \mathcal{P}(V_0) \quad \text{by} \quad \pi_1(S') = S', \quad \pi_{k+1}(X) = \bigcup_{Y \in X} \pi_k(Y),$$

so that $\pi = \pi_n$.

Theorem 2.22 (Embedding–Flattening Retraction)**.** *Let $\eta_n : V_0 \to \mathcal{P}^n(V_0)$ be the n-fold singleton embedding defined by $\eta_1(v) = \{v\}$ and $\eta_{k+1}(v) = \{\eta_k(v)\}$. Then*

$$\pi \circ \eta_n \;=\; \mathrm{id}_{V_0}.$$

Proof: By induction on k. For $k = 1$, $\pi_1(\eta_1(v)) = \pi_1(\{v\}) = \{v\}$. Since π_1 on singletons is the identity on subsets of V_0, and $\pi_{k+1}(\{\eta_k(v)\}) = \pi_k(\eta_k(v))$ by definition of π, the composition $\pi_{k+1} \circ \eta_{k+1}(v) = \pi_k(\eta_k(v))$. By the induction hypothesis $\pi_k \circ \eta_k(v) = v$. Hence $\pi \circ \eta_n(v) = v$ for all $v \in V_0$. □

Theorem 2.23 (Flattening Surjectivity on Hyperedges)**.** *The flattening map π induces a surjection*

$$\pi : \mathcal{E} \;\longrightarrow\; \mathcal{E}_{\text{flat}} \;=\; \big\{\pi(e) : e \in \mathcal{E}\big\} \;\subseteq\; \mathcal{P}(V_0).$$

Proof: By definition $\mathcal{E} \subseteq \mathcal{P}^n(V_0)$. Every hyperedge in the flattened network $\mathcal{E}_{\text{flat}}$ arises as $\pi(e)$ for some $e \in \mathcal{E}$. Thus π is onto $\mathcal{E}_{\text{flat}}$. □

Theorem 2.24 (Union and Intersection Preservation)**.** *For any $e, f \in \mathcal{E}$,*

$$\pi(e \cup f) = \pi(e) \,\cup\, \pi(f), \quad \pi(e \cap f) = \pi(e) \,\cap\, \pi(f).$$

Proof: By the recursive definition of π, unions and intersections commute with iterated unions. Concretely, at each level k, $\pi_{k+1}(X \cup Y) = \bigcup_{Z \in X \cup Y} \pi_k(Z) = \bigcup_{Z \in X} \pi_k(Z) \cup \bigcup_{Z \in Y} \pi_k(Z) = \pi_{k+1}(X) \cup \pi_{k+1}(Y)$, and similarly for intersections. Induction on k yields the result for $k = n$. □

Theorem 2.25 (Flow Preservation under Flattening)**.** *Suppose $f : \mathcal{E} \to \mathbb{R}_{\geq 0}$ is a feasible multicast flow in $\mathcal{H}^{(n)}$ for a demand (S, T, d), satisfying*

$$\sum_{e \ni X} f(e) - \sum_{e \ni X} f(e) = \begin{cases} d, & X \in S, \\ -d, & X \in T, \\ 0, & \textit{otherwise}, \end{cases} \qquad \sum_{e \in \mathcal{E}} f(e) \leq \kappa(e).$$

Then $f_{\text{flat}} : \mathcal{E}_{\text{flat}} \to \mathbb{R}_{\geq 0}$ defined by

$$f_{\text{flat}}(e') = \sum_{e:\, \pi(e) = e'} f(e)$$

is a feasible flow in the flattened hypernetwork $(V_0, \mathcal{E}_{\text{flat}}, \kappa', \delta')$ of the same demand.

Proof: For each base node $v \in V_0$, flow-conservation holds because every superhyperedge e with $\pi(e) = e'$ that contains the singleton $\eta_n(v)$ contributes $f(e)$ to both the aggregated inflow and outflow at v, mirroring the superhypernetwork balance. Capacity constraints translate as $\sum_{e' := \pi(e)} f_{\text{flat}}(e') = \sum_e f(e) \leq \kappa(e) \leq \kappa'(e')$ by defining $\kappa'(e') = \sum_{e : \pi(e) = e'} \kappa(e)$. Hence f_{flat} is feasible. □

Theorem 2.26 (Minimum Cut Duality)**.** *In the flattened hypernetwork, the value of the minimum S-T cut equals the value of the minimum cut in $\mathcal{H}^{(n)}$ under capacity aggregation:*

$$\min_{C \subseteq \mathcal{E}_{\text{flat}}} \sum_{e' \in C} \kappa'(e') \;=\; \min_{D \subseteq \mathcal{E}} \sum_{e \in D} \kappa(e).$$

Proof: Every cut $D \subseteq \mathcal{E}$ in the n-superhypernetwork induces a cut $\pi(D) \subseteq \mathcal{E}_{\text{flat}}$ in the flattened network with aggregated capacity $\sum_{e' \in \pi(D)} \kappa'(e') = \sum_{e \in D} \kappa(e)$. Conversely, any cut $C \subseteq \mathcal{E}_{\text{flat}}$ lifts to $D = \bigcup_{e' \in C} \{e : \pi(e) = e'\} \subseteq \mathcal{E}$ of the same capacity. Hence the minima coincide. □

3|Conclusion and Future Tasks

In this paper, we examined the mathematical definitions, structural properties, and practical examples of the *Telecommunications HyperNetwork* and the *Telecommunications SuperHyperNetwork*, which extend the classical Telecommunications Network to higher-order and hierarchical communication models.

Looking ahead, we anticipate conducting computational experiments and exploring real-world applications of these models. Furthermore, we plan to investigate extensions of the concepts introduced here to other "fuzzy-style" graph frameworks, such as Fuzzy Graphs [29, 30], Intuitionistic Fuzzy Graphs [31, 32, 33], Neutrosophic Graphs [34, 35, 36, 37], and Plithogenic Graphs [38, 39].

Funding

This study did not receive any financial or external support from organizations or individuals.

Acknowledgments

We extend our sincere gratitude to everyone who provided insights, inspiration, and assistance throughout this research. We particularly thank our readers for their interest and acknowledge the authors of the cited works for laying the foundation that made our study possible. We also appreciate the support from individuals and institutions that provided the resources and infrastructure needed to produce and share this paper. Finally, we are grateful to all those who supported us in various ways during this project.

Ethical Approval

As this research is entirely theoretical in nature and does not involve human participants or animal subjects, no ethical approval is required.

Data Availability

This research is purely theoretical, involving no data collection or analysis. We encourage future researchers to pursue empirical investigations to further develop and validate the concepts introduced here.

Research Integrity

The authors hereby confirm that, to the best of their knowledge, this manuscript is their original work, has not been published in any other journal, and is not currently under consideration for publication elsewhere at this stage.

Disclaimer (Note on Computational Tools)

No computer-assisted proof, symbolic computation, or automated theorem proving tools (e.g., Mathematica, SageMath, Coq, etc.) were used in the development or verification of the results presented in this paper. All proofs and derivations were carried out manually and analytically by the authors.

Disclaimer (Limitations and Claims)

The theoretical concepts presented in this paper have not yet been subject to practical implementation or empirical validation. Future researchers are invited to explore these ideas in applied or experimental settings. Although every effort has been made to ensure the accuracy of the content and the proper citation of sources, unintentional errors or omissions may persist. Readers should independently verify any referenced materials.

To the best of the authors' knowledge, all mathematical statements and proofs contained herein are correct and have been thoroughly vetted. Should you identify any potential errors or ambiguities, please feel free to contact the authors for clarification.

The results presented are valid only under the specific assumptions and conditions detailed in the manuscript. Extending these findings to broader mathematical structures may require additional research. The opinions and conclusions expressed in this work are those of the authors alone and do not necessarily reflect the official positions of their affiliated institutions.

Competing interests

Author has declared that no competing interests exist.

Consent to Publish declaration

The author approved to Publish declarations.

References

[1] Reinhard Diestel. Graph theory 3rd ed. *Graduate texts in mathematics*, 173(33):12, 2005.

[2] Jonathan L Gross, Jay Yellen, and Mark Anderson. *Graph theory and its applications*. Chapman and Hall/CRC, 2018.

[3] Claude Berge. *Hypergraphs: combinatorics of finite sets*, volume 45. Elsevier, 1984.

[4] Derun Cai, Moxian Song, Chenxi Sun, Baofeng Zhang, Shenda Hong, and Hongyan Li. Hypergraph structure learning for hypergraph neural networks. In *IJCAI*, pages 1923–1929, 2022.

[5] Song Feng, Emily Heath, Brett Jefferson, Cliff Joslyn, Henry Kvinge, Hugh D Mitchell, Brenda Praggastis, Amie J Eisfeld, Amy C Sims, Larissa B Thackray, et al. Hypergraph models of biological networks to identify genes critical to pathogenic viral response. *BMC bioinformatics*, 22(1):287, 2021.

[6] Takaaki Fujita and Prem Kumar Singh. Hyperfuzzy graph and hyperfuzzy hypergraph. *Journal of Neutrosophic and Fuzzy Systems (JNFS)*, 10(01):01–13, 2025.

[7] Florentin Smarandache. *Introduction to the n-SuperHyperGraph-the most general form of graph today*. Infinite Study, 2022.

[8] Eduardo Martín Campoverde Valencia, Jessica Paola Chuisaca Vásquez, and Francisco Ángel Becerra Lois. Multineutrosophic analysis of the relationship between survival and business growth in the manufacturing sector of azuay province, 2020–2023, using plithogenic n-superhypergraphs. *Neutrosophic Sets and Systems*, 84:341–355, 2025.

[9] Julio Cesar Méndez Bravo, Claudia Jeaneth Bolanos Piedrahita, Manuel Alberto Méndez Bravo, and Luis Manuel Pilacuan-Bonete. Integrating smed and industry 4.0 to optimize processes with plithogenic n-superhypergraphs. *Neutrosophic Sets and Systems*, 84:328–340, 2025.

[10] Takaaki Fujita. Modeling hierarchical systems in graph signal processing, electric circuits, and bond graphs via hypergraphs and superhypergraphs. *Journal of Engineering Research and Reports*, 27(5):542, 2025.

[11] Florentin Smarandache. Foundation of superhyperstructure & neutrosophic superhyperstructure. *Neutrosophic Sets and Systems*, 63(1):21, 2024.

[12] Alain Bretto. Hypergraph theory. *An introduction. Mathematical Engineering. Cham: Springer*, 1, 2013.

[13] Florentin Smarandache. *Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra*. Infinite Study, 2020.

[14] Takaaki Fujita. Exploration of graph classes and concepts for superhypergraphs and n-th power mathematical structures. 2025.

[15] John Edward Flood. *Telecommunication networks*. IET, 1997.

[16] Mohamed El-Sayed and Jeffrey Jaffe. A view of telecommunications network evolution. *IEEE Communications Magazine*, 40(12):74–81, 2002.

[17] Mischa Schwartz. *Telecommunication networks: protocols, modeling and analysis*. Addison-Wesley Longman Publishing Co., Inc., 1986.

[18] Victor S Frost and Benjamin Melamed. Traffic modeling for telecommunications networks. *IEEE Communications magazine*, 32(3):70–81, 2002.

[19] Krzysztof Pawlikowski, H-DJ Jeong, and J-SR Lee. On credibility of simulation studies of telecommunication networks. *IEEE Communications magazine*, 40(1):132–139, 2002.

[20] Carmen Egido. Video conferencing as a technology to support group work: a review of its failures. In *Proceedings of the 1988 ACM conference on Computer-supported cooperative work*, pages 13–24, 1988.

[21] Kyle MacMillan, Tarun Mangla, James Saxon, and Nick Feamster. Measuring the performance and network utilization of popular video conferencing applications. In *Proceedings of the 21st ACM Internet Measurement Conference*, pages 229–244, 2021.

[22] Scott Firestone, Thiya Ramalingam, and Steve Fry. *Voice and video conferencing fundamentals*. Cisco Press, 2007.

[23] Ibrahim Nadir, Haroon Mahmood, and Ghalib Asadullah. A taxonomy of iot firmware security and principal firmware analysis techniques. *International Journal of Critical Infrastructure Protection*, 38:100552, 2022.

[24] Taimur Bakhshi, Bogdan Ghita, and Ievgeniia Kuzminykh. A review of iot firmware vulnerabilities and auditing techniques. *Sensors*, 24(2):708, 2024.

[25] Joerg Walz and Brian Neil Levine. A hierarchical multicast monitoring scheme. In *Proceedings of NGC 2000 on Networked group communication*, pages 105–116, 2000.

[26] R Venkateswaran, CS Raghavendra, Xiaoqiang Chen, and VP Kumar. Hierarchical multicast routing in atm networks. In *Proceedings of ICC/SUPERCOMM'96-International Conference on Communications*, volume 3, pages 1690–1694. IEEE, 1996.

[27] Tilman Wolf and Sumi Y Choi. Aggregated hierarchical multicast for active networks. In *2001 MILCOM Proceedings Communications for Network-Centric Operations: Creating the Information Force (Cat. No. 01CH37277)*, volume 2, pages 899–904. IEEE, 2001.

[28] Tony H Grubesic and Morton E O'Kelly. Using points of presence to measure accessibility to the commercial internet. *The Professional Geographer*, 54(2):259–278, 2002.

[29] TM Nishad, Talal Ali Al-Hawary, and B Mohamed Harif. General fuzzy graphs. *Ratio Mathematica*, 47, 2023.

[30] Azriel Rosenfeld. Fuzzy graphs. In *Fuzzy sets and their applications to cognitive and decision processes*, pages 77–95. Elsevier, 1975.

[31] Sankar Sahoo and Madhumangal Pal. Product of intuitionistic fuzzy graphs and degree. *Journal of Intelligent & Fuzzy Systems*, 32(1):1059–1067, 2017.

[32] M. G. Karunambigai, R. Parvathi, and R. Buvaneswari. Arc in intuitionistic fuzzy graphs. *Notes on Intuitionistic Fuzzy Sets*, 17:37–47, 2011.

[33] Hossein Rashmanlou, Sovan Samanta, Madhumangal Pal, and Rajab Ali Borzooei. Intuitionistic fuzzy graphs with categorical properties. *Fuzzy information and Engineering*, 7(3):317–334, 2015.

[34] D Ajay, P Chellamani, G Rajchakit, N Boonsatit, and P Hammachukiattikul. Regularity of pythagorean neutrosophic graphs with an illustration in mcdm. *AIMS mathematics*, 7(5):9424–9442, 2022.

[35] Said Broumi, Mohamed Talea, Assia Bakali, and Florentin Smarandache. Complex neutrosophic graphs of type. *Collected Papers. Volume VI: On Neutrosophic Theory and Applications*, page 204, 2022.

[36] Naveed Yaqoob and Muhammad Akram. *Complex neutrosophic graphs*. Infinite Study, 2018.

[37] Rupkumar Mahapatra, Sovan Samanta, Madhumangal Pal, and Qin Xin. Link prediction in social networks by neutrosophic graph. *International Journal of Computational Intelligence Systems*, 13(1):1699–1713, 2020.

[38] Takaaki Fujita. Claw-free graph and at-free graph in fuzzy, neutrosophic, and plithogenic graphs. *Information Sciences with Applications*, 5:40–55, 2025.

[39] Takaaki Fujita. Short note of extended hyperplithogenic sets and general extended plithogenic graphs. *Advancing Uncertain Combinatorics through Graphization, Hyperization, and Uncertainization: Fuzzy, Neutrosophic, Soft, Rough, and Beyond*, page 57, 2025.